ANALYSIS OF SWEPT TIME AND MODAL PARAMETER EFFECT ON FRF'S MAGNITUDE ERROR OF SDOF SYSTEM USING LINEAR SWEPT-SINE EXCITATION

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Abstract

Random and swept-sine excitations are the most commonly used excitations in FRF (Frequency Response Function) measurement. There are various type of swept-sine excitation can be used, however, linear swept-sine is often used in FRF measurement. Linear swept-sine excitation is a sinusoidal excitation of which its frequencies change linearly within time. This paper presents swept time and modal parameter effect on FRF's magnitude error of SDOF (Single Degree of Freedom) system using linear swept-sine excitation. FRF's magnitude error is focused at the peak of FRF's magnitude error (system's resonant frequency). Based on analysis study, the peak of FRF's magnitude error of SDOF system is a function of swept time and system's modal parameter (resonant frequency and damping ratio).

Keywords: FRF, linear swept-sine excitation, SDOF system, the peak of FRF's magnitude error.

Abstrak

Eksitasi random dan swept-sine merupakan eksitasi yang biasa digunakan pada pengukuran FRF (Fungsi Respon Frekuensi). Ada beberapa tipe dari eksitasi swept-sine yang dapat digunakan, bagaimanapun, eksitasi linear swept-sine paling sering digunakan pada pengukuran FRF. Eksitasi linear swept-sine adalah sebuah eksitasi sinusoidal yang mana kandungan frekuensinya berubah secara linear terhadap waktu. Makalah ini memaparkan pengaruh waktu swept dan parameter modal terhadap kesalahan magnitudo FRF dari Sistem 1-DK (Satu Derajat Kebebasan) dengan eksitasi linear swept-sine. Kesalahan magnitudo FRF difokuskan pada kesalahan puncak magnitudo FRF (frekuensi resonansi sistem). Berdasarkan studi analisis, kesalahan puncak magnitudo FRF Sistem 1-DK merupakan fungsi waktu swept dan parameter modal (frekuensi resonansi dan rasio redaman).

Kata kunci: FRF, eksitasi linear swept-sine, Sistem 1-DK, kesalahan puncak magnitudo FRF.

I. INTRODUCTION

FRF (Frequency Response Function) measurement is conducted to depict dynamic characteristic of a system. In the FRF measurement, the system is vibrated by using exciter. Both excitation force and system's vibration response are measured concurrently. By using signal's frequencies analyzer, both measured signals are transformed into force's frequencies spectrum and response frequencies spectrum respectively. Ratio of response frequencies spectrum magnitude to force's frequencies spectrum magnitude is known as FRF's magnitude of the system (McConnell, 1995).

The most commonly used exciter in the FRF measurement is impact hammer or shaker. For shaker excitation, some equipment's are needed. They are signal generator, power amplifier, shaker, and stinger (Peres et.al, 2010; Cloutier et.al, 2009; Füllekrug et.al., 2008). The advantage of using shaker's excitation is its frequencies can be controlled very well.

There are various type of shaker's excitations can be used, however, swept-sine excitation is the most common type of excitation in the FRF measurement (Zhuge, 2009; Peeters et.al., 2008; Climent, 2007, Göge et.al., 2007; Pauwels et.al., 2006; Schwarz, 1999). Swept-sine excitation is a sinusoidal excitation of which its frequencies change within time or depend on swept function. If swept-sine excitation has linear swept function, then, it is called with linear swept-sine. It is a kind of swept-sine excitation that often be used in FRF measurement (Orlando et.al., 2008; Gloth et.al., 2004(a), 2004(b); Baoliang et.al., 2003; Haritos, 2002).

FRF's magnitude of a system at certain frequency span can be obtained from FRF measurement using linear swept-sine excitation for different swept time. The swept time is duration that needed for exciting the system in FRF measurement at certain frequency span. For tested system as well as SDOF (Single Degree of Freedom) system, it can be classified based on its modal parameter (resonant frequency and damping ratio). To better understand swept time and modal parameter effect on FRF's magnitude error of SDOF system, an analysis study is presented in this paper. Here, frequency span value is limited only 40 Hz. This value is chosen because the first resonant frequency of the system often lies under it. FRF's magnitude error of the system is focused at the peak of FRF's magnitude error (system's resonant frequency).

II. METHODOLOGY

Analysis study to understand swept time and modal parameter effect on FRF's magnitude error of SDOF system using linear swept-sine excitation is conducted through a numerical simulation. The simulation contains some stages in flowchart as shown in Figure 1.

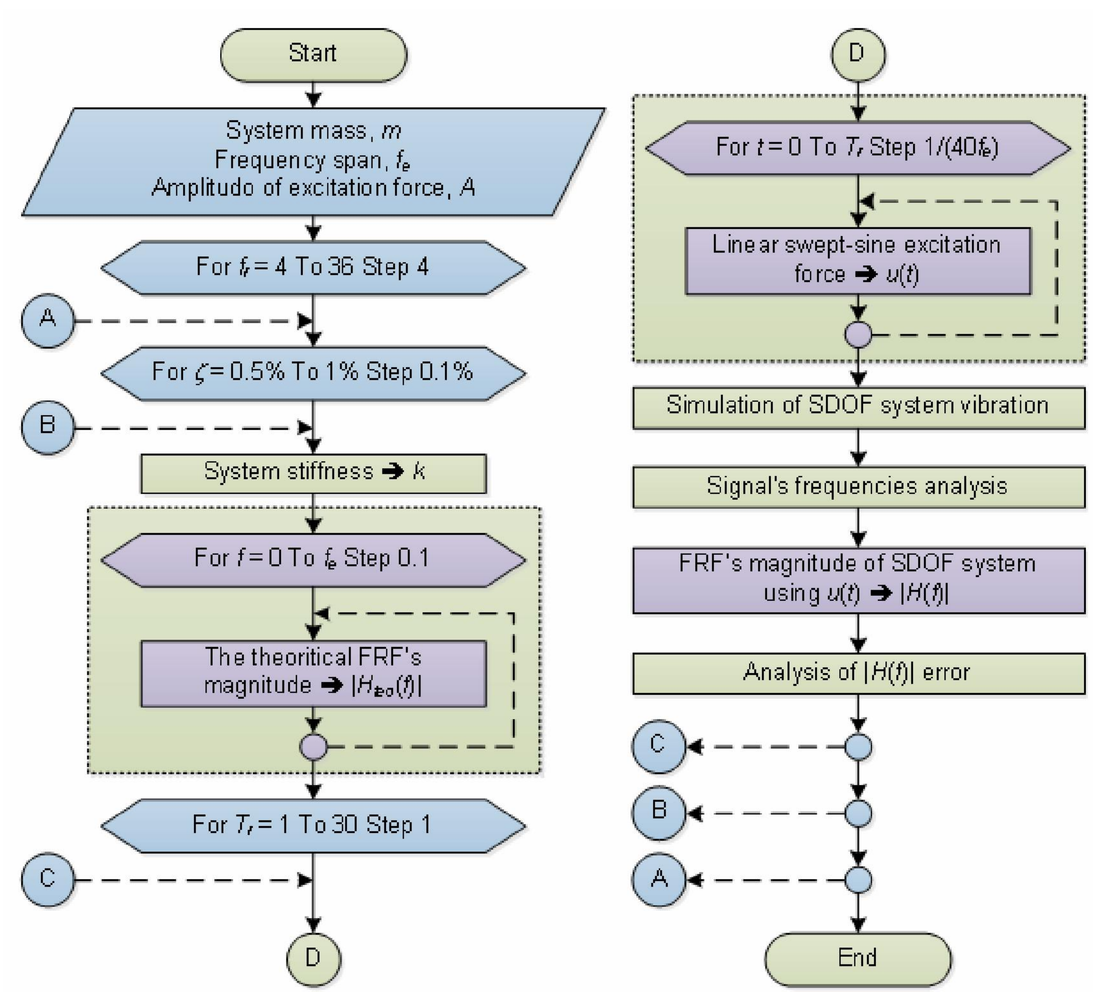

Figure 1. Flowchart of analysis study to understand swept time and modal parameter effect on FRF's magnitude error of SDOF system using linear swept-sine excitation.

System mass m, frequency span f_e, amplitude of excitation force A, resonant frequency f_r, damping ratio ζ, and swept time T_r are simulation's input parameters. For each of various resonant frequencies f_r, system stiffness k and system damping c respectively are

$$k = \frac{4\pi^2 m f_r^2}{(1-2\zeta^2)} \tag{1}$$

$$c = 2\zeta\sqrt{mk} \tag{2}$$

Then, the theoretical FRF's magnitude of SDOF system $|H_{teo}(f)|$ is obtained.

$$|H_{teo}(f)| = \frac{1/k}{\sqrt{\left[1-\left(\frac{f\sqrt{1-2\zeta^2}}{f_r}\right)^2\right]^2+\left(2\zeta\frac{f\sqrt{1-2\zeta^2}}{f_r}\right)^2}} \tag{3}$$

The linear swept-sine excitation $u(t)$ has starting frequency f_0, ending frequency that equal to span frequency f_e, swept-time T_r, and amplitude A (Yanto et. al., 2012a). The $u(t)$ is expressed with

$$u(t) = A\sin\left\{\frac{\pi}{2}(f_e - f_0)\frac{t^2}{T_r} + 2\pi f_0 t\right\} \tag{4}$$

If a SDOF system is excited by using $u(t)$, then, it is modeled as the impulse response function $h(t)$. Figure 2 shows the $h(t)$ input-output relationship (Yanto et. al., 2012b).

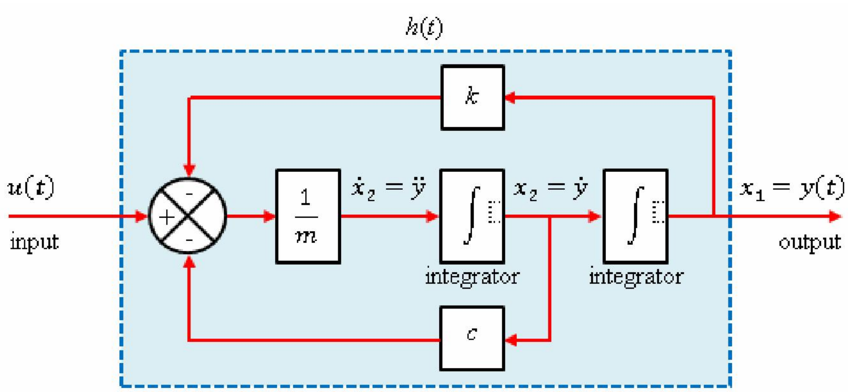

Figure 2. The $h(t)$ input-output relationship.

The system state $x(t)$ and output $y(t)$ can be written as follow:

$$\begin{Bmatrix} x_1(t_{i+1}) \\ x_2(t_{i+1}) \end{Bmatrix} = \frac{e^{-a\Delta t}-e^{-b\Delta t}}{b-a}\begin{bmatrix} 2\zeta\sqrt{\frac{k}{m}} - \frac{ae^{-a\Delta t}-be^{-b\Delta t}}{e^{-a\Delta t}-e^{-b\Delta t}} & 1 \\ -\frac{k}{m} & -\frac{ae^{-a\Delta t}-be^{-b\Delta t}}{e^{-a\Delta t}-e^{-b\Delta t}} \end{bmatrix}\begin{Bmatrix} x_1(t_i) \\ x_2(t_i) \end{Bmatrix}$$
$$+\frac{1}{m(b-a)}\begin{Bmatrix} -\frac{1}{a}\left(e^{-a\Delta t}-1\right) + -\frac{1}{a}\left(e^{-a\Delta t}-1\right) \\ e^{-a\Delta t}-e^{-b\Delta t} \end{Bmatrix} u(t_i) \tag{5}$$

$$y(t) = x_1(t_i) \quad ; \quad 0 \le t_i \le T_r \tag{6}$$

Where:

$$i = 0, 1, 2, \dots, \left\lfloor \frac{T_r}{\Delta t} \right\rfloor$$

$$a = \zeta\sqrt{\frac{k}{m}} + j\sqrt{\frac{k}{m}(1-\zeta^2)}$$

$$b = \zeta\sqrt{\frac{k}{m}} - j\sqrt{\frac{k}{m}(1-\zeta^2)}$$

$\Delta t = \frac{1}{40 f_e}$: Sampling period

$\begin{Bmatrix} x_1(t_0=0) \\ x_2(t_0=0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$: Initial conditions

Equations (5) and (6) are the equations for simulating SDOF system vibration using linear swept-sine excitation. The simulation of SDOF system vibration is conducted to obtain both numerically system's vibration response $y(t)$ signal. Then, both $u(t)$ and $y(t)$ signals are transformed into force's frequencies spectrum $U(f)$ and response frequencies spectrum $Y(f)$ by using Fourier Transform Method respectively as shown in Figure 3. The FRF is obtained by

$$H(f) = \frac{G_{uy}(f)}{G_{uu}(f)} \tag{7}$$

Where:

$G_{uy}(f) = U^*(f)Y(f)$: The Cross Power Spectrum between $u(t)$ and $y(t)$.

$G_{uu}(f) = U^*(f)U(f)$: The Auto Power Spectrum of $u(t)$.

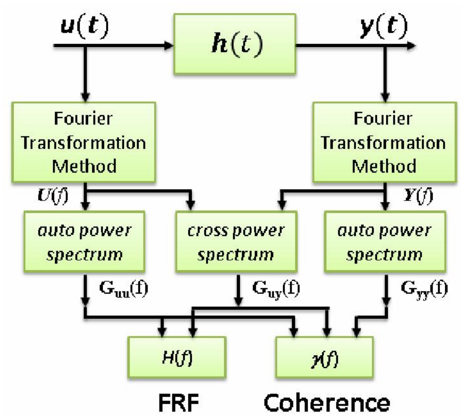

Figure 3. The dual channels frequency analysis.

The FRF's magnitude error is focused at system's resonant frequency that be determined by

$$E_a(f_r) = \frac{|H_{teo}(f_r)| - |H(f_r)|}{|H_{teo}(f_r)|} \cdot 100\% \tag{8}$$

III. RESULTS AND DISCUSSION

Value of input parameters in analysis study of swept time and modal parameter effect on FRF's magnitude error of SDOF system are shown in Table 1.

Table 1. Value of input parameters.

Parameter	Value	Unit
m	1	kg
f_e	40	Hz
A	1	N
f_r	4 To 36 Step 4	Hz
ζ	0.5 To 1 Step 0.1	%
T_r	1 To 30 Step 1	s

Figure 4 shows an example of the theoretical FRF's magnitude of SDOF system where m = 1 kg, f_r = 16 Hz, ζ = 1%, and f_e = 40 Hz are known.

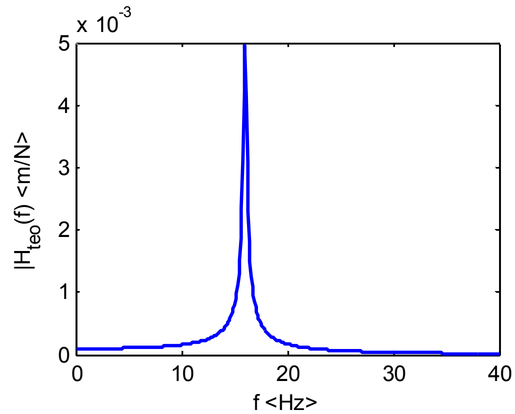

Figure 4. The theoretical FRF's magnitude of SDOF system where m = 1 kg, f_r = 16 Hz, ζ = 1%, and f_e = 40 Hz are known.

If linear swept-sine excitation has f_0 = 0 Hz, f_e = 40 Hz, and T_r = 4 s, so, it can be described as shown in Figure 5.

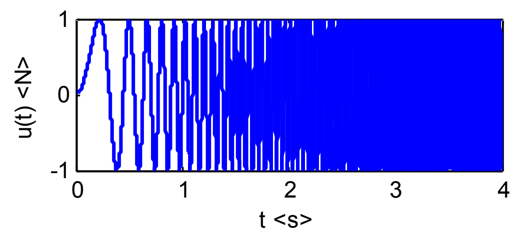

Figure 5. Linear swept-sine excitation where f_0 = 0 Hz, f_e = 40 Hz, and T_r = 4.

The linear swept-sine excitation in Figure 5 is applied on the SDOF system of which m = 1 kg, f_r = 16 Hz, and ζ = 1% would cause system's vibration response as shown in Figure 6.

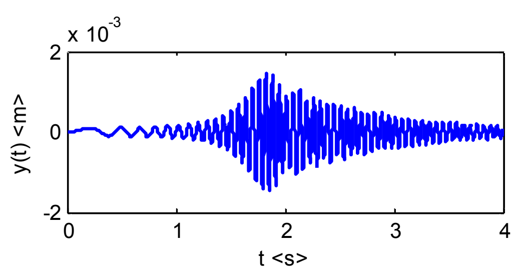

Figure 6. Vibration response of SDOF system (m = 1 kg, f_r = 16 Hz, and ζ = 1%) on linear swept-sine excitation (f_0 = 0 Hz, f_e = 40 Hz, and T_r = 4).

The FRF's magnitude of of SDOF system (m = 1 kg, f_r = 16 Hz, and ζ = 1%) using linear swept-sine excitation (f_0 = 0 Hz, f_e = 40 Hz, and T_r = 4) is compared with its theoretical FRF's magnitude can be illustrated in Figure 7.

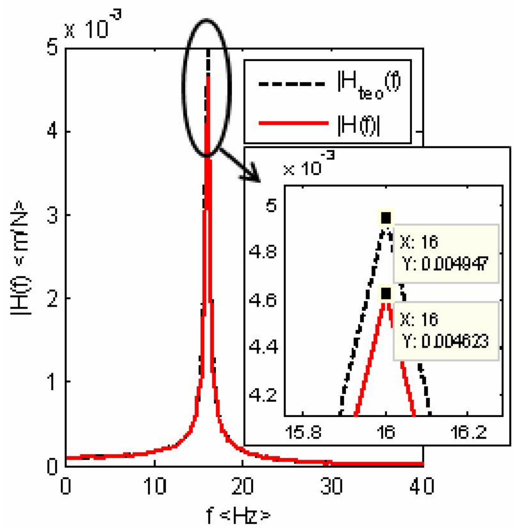

Figure 7. The FRF's magnitude of SDOF system using linear swept-sine excitation is compared with its theoretical FRF's magnitude.

For SDOF system of which value of its resonant frequency f_r and damping ratio ζ are varied as in Table 1, the peak of FRF's magnitude error $E_\alpha(f_r)$ are shown in Figure 8. This error is a function of swept time and system's modal parameter (resonant frequency and damping ratio) that can be expressed by

$$E_\alpha(f_r) = f(T_r, f_r, \zeta) \quad (9)$$

Values of $E_\alpha(f_r)$ are more influenced by swept time T_r for each of system's modal parameter. If linear swept-sine excitation is applied to SDOF system with swept time T_r equal to or large than 23 s, then, obtained $E_\alpha(f_r)$ less than 10 % as shown in Figure 9.

IV. CONCLUSION

FRF's magnitude error of SDOF systems using linear swept-sine excitation are effected by swept time and system's modal parameter. FRF's magnitude error less than 10 % can be obtained by using linear swept-sine excitation that has swept time equal to or large than 23 seconds for each of system's modal parameter.

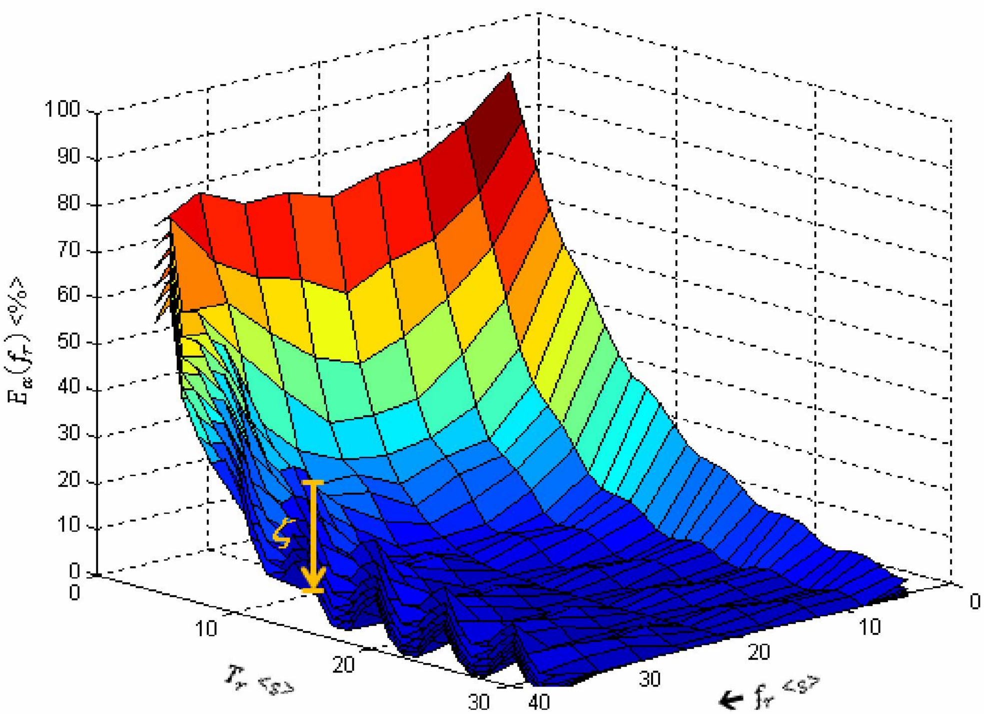

Figure 8. The FRF's magnitude error SDOF system using linear swept-sine excitation to its theoretical FRF's magnitude.

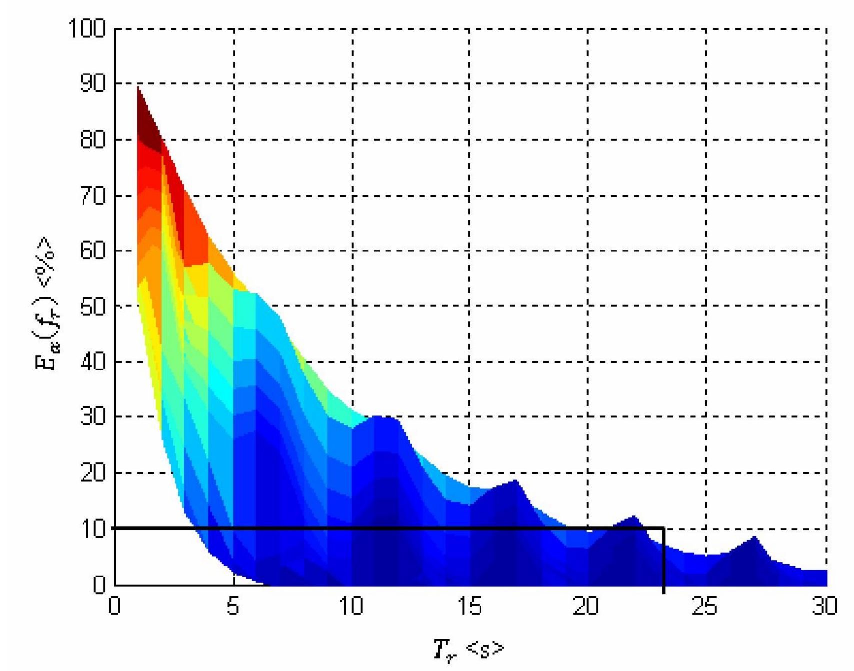

Figure 9. Swept time T_r effect on values of $E_\alpha(f_r)$ for each of system's modal parameter.

This analysis study can be used as a reference in FRF measurement using linear swept-sine excitation to avoid large error of FRF's magnitude of the testing system experimentally.

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